

APPROXIMATE FUNCTIONS FOR THE FAST CALCULATION OF LIGHT-WATER PROPERTIES AT SATURATION

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Abstract—For thermalhydraulic systems analysis, linear interpolation algorithms are commonly used for the calculation of thermodynamic properties. However, these algorithms can use a substantial amount of computer time and memory. An alternative to this approach suggested in the past is the use of approximation formulas. Such formulas for the calculation of the thermodynamic properties of light water for saturation conditions are presented here, based on the 1984 *NBS/NRC Steam Tables*. The range of these approximations is from below 1b to just below the critical point (22.055 MPa) with a deviation from tabulated values of not more than 0.22%. The formulas were determined by the method of least squares, enabling a minimization of deviations from the line of best fit and the fitting of functions simple enough to be used with programmable calculators, as well as microcomputers. In addition to the rapid calculation of the properties, the simple curve fits are instrumental in the development of the rate form of the equation of state.

Since any given property cannot be accurately fitted over the entire pressure range with a single simple expression, the pressure range was split into subranges. Special care was taken to ensure that the slopes of the curve fits were continuous across the boundaries since discontinuities in the slopes of the property tables can cause instabilities and failure of search algorithms in typical computer codes.

Key Words: water properties, curve fit, fast, saturation.

INTRODUCTION

In the analysis of flow systems, the thermodynamic properties are usually calculated by linear interpolation algorithms applied to thermodynamic tables stored on computer. However, the storage of these steam tables can occupy a large amount of computer memory. As well, the linear interpolation algorithms require a searching algorithm to select the proper numbers from the tables. Thus the use of such algorithms can considerably tax computer running time. An alternative to this method is the employment of approximate formulas, similar to those described by Firla (1984), which can rapidly compute the value of a property with satisfactory accuracy for the purpose of system analysis.

This paper concentrates on the thermodynamic properties of light water for saturation conditions. At saturation, the temperature can be expressed as a function of pressure only. Therefore, we can represent the properties by a number of simple functions containing one independent variable: pressure.

In addition to the direct calculation of thermodynamic properties, these approximation functions can be applied to the determination of the rate form of the equation of state (Garland & Sollychin 1988).

APPROXIMATION METHOD

The approach taken in developing the correlations minimized the deviations from the reference steam tables. To maintain a high accuracy it was necessary to subdivide the range of pressure variation into several regions. The simple functions used were fitted to the data by the method of least squares, as discussed in the following section.

As system codes often require the slopes of the properties, the fits to the steam table by a set of approximation functions, were required to exhibit a continuous first derivative across the entire range of pressure.

Least-squares method

We represent a set of n data points by some relationship $y = f(x)$, containing p unknown parameters a_1, a_2, \dots, a_p , the deviations or residuals are given by

$$D_i = f(x_i) - y_i. \quad [1]$$

The sum of the squares of the deviations,

$$S = \sum_{i=1}^n D_i^2 = \sum_{i=1}^n [f(x_i) - y_i]^2, \quad [2]$$

is a function of a_1, a_2, \dots, a_p . The parameters are determined such that S is a minimum ($dS/da_1 = 0, dS/da_2 = 0, \dots, dS/da_p = 0$).

If we take $y = f(x)$ to be a linear function ($y = a_1 + a_2x$), the residuals are $D_i = (a_1 + a_2x_i) - y_i$, so that

$$S = (a_1 + a_2x_1 - y_1)^2 + (a_1 + a_2x_2 - y_2)^2 + \dots + (a_1 + a_2x_n - y_n)^2. \quad [3]$$

On differentiating S with respect to a_1 and a_2 , two equations are obtained:

$$\frac{dS}{da_1} = 2(a_1 + a_2x_1 - y_1) + 2(a_1 + a_2x_2 - y_2) + \dots + 2(a_1 + a_2x_n - y_n) = 0$$

and

$$\frac{dS}{da_2} = 2(x_1)(a_1 + a_2x_1 - y_1) + 2(x_2)(a_1 + a_2x_2 - y_2) + \dots + 2(x_n)(a_1 + a_2x_n - y_n) = 0 \quad [4]$$

Dividing by two and collecting the coefficients of a_1 and a_2 , we get

$$na_1 + \left(\sum_{i=1}^n x_i \right) a_2 = \sum_{i=1}^n y_i$$

and

$$\left(\sum_{i=1}^n x_i \right) a_1 + \left(\sum_{i=1}^n x_i^2 \right) a_2 = \sum_{i=1}^n x_i y_i. \quad [5]$$

Similarly, for a second-order polynomial (quadratic equation):

$$4a_1 + \left(\sum_{i=1}^n x_i \right) a_2 + \left(\sum_{i=1}^n x_i^2 \right) a_3 = \sum_{i=1}^n y_i, \quad [6a]$$

$$\left(\sum_{i=1}^n x_i \right) a_1 + \left(\sum_{i=1}^n x_i^2 \right) a_2 + \left(\sum_{i=1}^n x_i^3 \right) a_3 = \sum_{i=1}^n x_i y_i, \quad [6b]$$

and

$$\left(\sum_{i=1}^n x_i^2 \right) a_1 + \left(\sum_{i=1}^n x_i^3 \right) a_2 + \left(\sum_{i=1}^n x_i^4 \right) a_3 = \sum_{i=1}^n x_i^2 y_i. \quad [6c]$$

These equations can be solved for a_1, a_2 and a_3 to give the function $y = a_1 + a_2x + a_3x^2$. Higher order polynomials may also be fitted in this manner, of course.

Correlations are often described by a correlation constant, r . This number expresses the strength and direction of the correlation and can vary from $+1.00$ to -1.00 . For positive correlations where an increase in one variable tends to lead to an increase in the other variable being considered, r is positive. For negative correlations where an increase in one variable tends to lead to a decrease in the other, r is negative. The largest magnitude of r is 1.00 which represents a perfect correlation. Thus the closer the points in a plot of the two variables come to falling on the line of best fit, the nearer r will be to $+1.00$ or -1.00 . The following section describes different types of functions that can be determined using the method of least squares. The correlation constant can be used as a way to compare each function and to see if the range being fitted is too large to obtain a high enough accuracy with the steam tables.

Table 1. Transforming functions into a linear form

Function	Operations on data	Linear form
$y = \frac{a}{x} + b$	$x \Rightarrow \frac{1}{x}$	$y = a\left(\frac{1}{x}\right) + b$
$y = ax^b$	$x \Rightarrow \log x$ $y \Rightarrow \log y$	$(\log y) = b(\log x) + \log a$
$y = a \exp(bx)$	$y \Rightarrow \log y$	$(\log y) = bx + \log a$
$y = a \log x + b$	$x \Rightarrow \log x$	$y = a(\log x) + b$

The correlation constant for $y = ax + b$ is calculated as

$$r = \frac{b \sum_{i=1}^n y_i + a \sum_{i=1}^n (x_i y_i) - \frac{\left(\sum_{i=1}^n y_i\right)^2}{n}}{\sum_{i=1}^n (y_i)^2 - \frac{\left(\sum_{i=1}^n y_i\right)^2}{n}} \tag{7}$$

Similarly, for $y = ax^2 + bx + c$,

$$r = \frac{c \sum_{i=1}^n y_i + b \sum_{i=1}^n (x_i y_i) + a \sum_{i=1}^n (x_i^2 y_i) - \frac{\left(\sum_{i=1}^n y_i\right)^2}{n}}{\sum_{i=1}^n (y_i)^2 - \frac{\left(\sum_{i=1}^n y_i\right)^2}{n}} \tag{8}$$

Using least squares for other functions

Data can be approximated by other functions which include power, exponential and logarithmic forms. The coefficients of each of these functions can be determined using [5] by altering the data, as shown in table 1, to transform the functions into a linear form. The power function and the exponential function are expected to be useful in producing an accurate curve fit of the thermodynamic properties. However, the method of least squares determines these functions such that $x = 0$ for $y = 0$. To obtain better accuracy with these curves, we can shift the data by adding to or subtracting from the x and y values. For example, figure 1(a) shows two curves passing

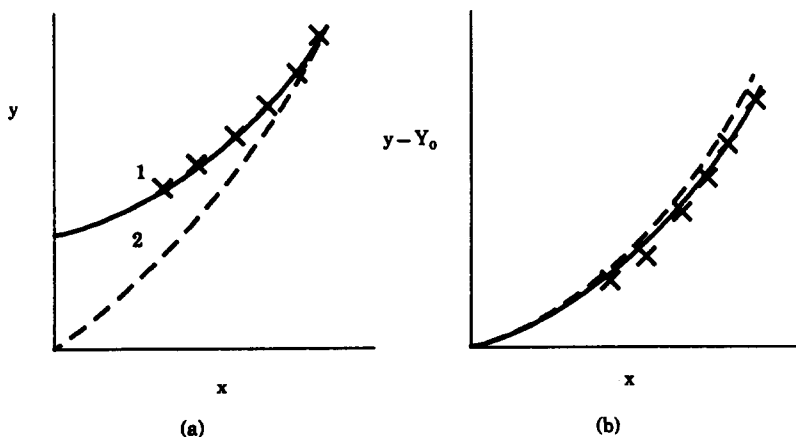


Figure 1. Example of data shifting. (a) The solid line represents the best-fit curve as determined by sight. The dashed line represents the best-fit power curve as determined by the method of least squares. (b) The y values in (a) have a constant value, Y_0 , subtracted from them such that the solid line passes through the origin. The dashed line is now more comparable to the solid line and is given by $y - Y_0 = ax^b$, where a and b are found using [5].

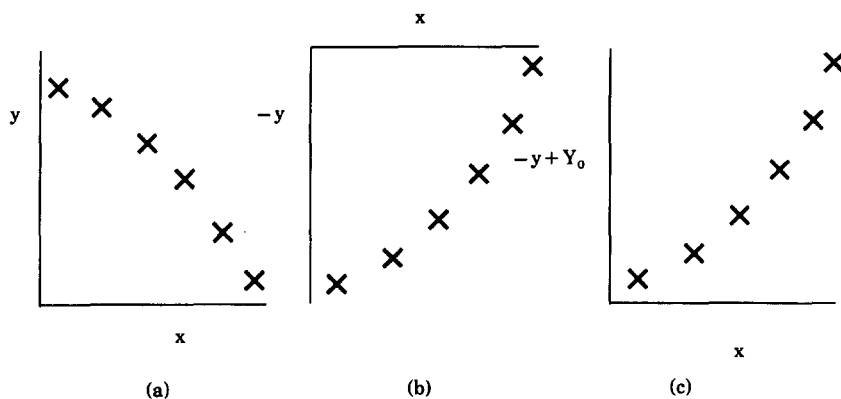


Figure 2. Dealing with negative sloping curves. (a) The actual data points are plotted. The slope of a curve passing through these points is negative. From figure 1, one can see that the shift that would produce the most accurate fit makes all $(y - Y_0)$ values negative in this case. (b) The negative y values are plotted here. The slope of a curve passing through these points is positive. (c) The negative y values are shifted upward by a constant, Y_0 , such that all $y'' = -y + Y_0$ are positive. The method of least squares can now be used for the power and exponential functions. For the power function, the equation determined by least squares is given by $y = Y_0 - ax^b$.

through a set of points. Curve 1 is the best fit, as determined by sight, and curve 2 is the best fit of a power function, as determined by the method of least squares, without shifting the data. Figure 1(b) shows the two curves after a constant, Y_0 , has been subtracted from each y value. We can see the effectiveness of a shifting of data. By comparing the correlation constants for different shifts, one can determine which shift gives the most accurate curve fit. The shifting of the data should be done before the operations described in table 1 are carried out.

In some circumstances, such as when there is a decrease in y for an increase in x , the necessary shifting of data would produce negative values either in the x or y direction. To avoid taking the log of a negative number we can fit the negative of the y values. Figure 2 demonstrates this procedure.

CORRELATIONS OF LIGHT-WATER THERMODYNAMIC PROPERTIES†

The following thermodynamic properties of light water at saturation were fitted to approximation functions: (1) specific volume/density, (2) specific enthalpy, (3) saturation temperature, (4) specific entropy, (5) specific heat and (6) dynamic viscosity. The reference source of data for all of these properties, with the exception of viscosity, is the *NBS/NRC Steam Tables* (Haar *et al.* 1984). The subroutines by Sokolnikoff & Redheffer (1966) were used for the calculation of the above properties.

These subroutines were also used in conjunction with the equation for viscosity given by White (1975/1983). This combination was shown to yield an adequate representation of viscosity by Kamgar-Parsi & Sengers (1982).

The set of functions for each property are listed along with their range of use and the worst accuracy encountered over this range. Figures 3–13 show the properties and the accuracy of the approximation, as calculated by

$$\text{accuracy} = \frac{Y_{\text{approx}} - Y_{\text{steam tables}}}{Y_{\text{steam tables}}} * 100 [\%]. \quad [9]$$

For all of the properties, the range of each function was chosen such that the accuracy is as small as possible and the first derivatives of two adjoining functions are equal at the point where they

†Program diskettes, containing the programs used in the determination of the approximation functions and in the reproduction of property tables, can be obtained from the first author. These diskettes are available in either PDP11 or IBM-PC format.

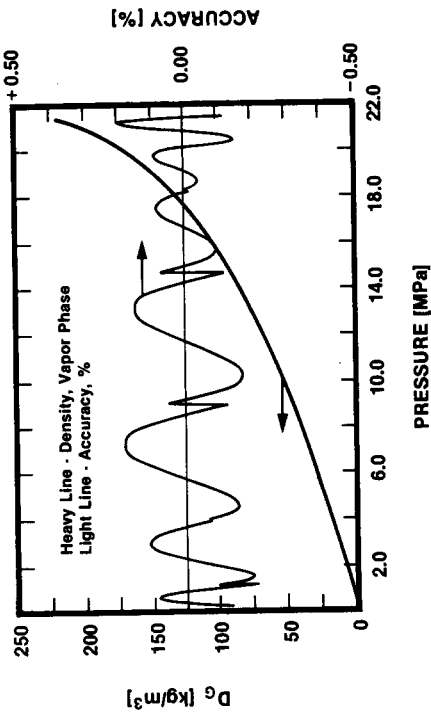


Figure 4. Density of the vapor phase at saturation.

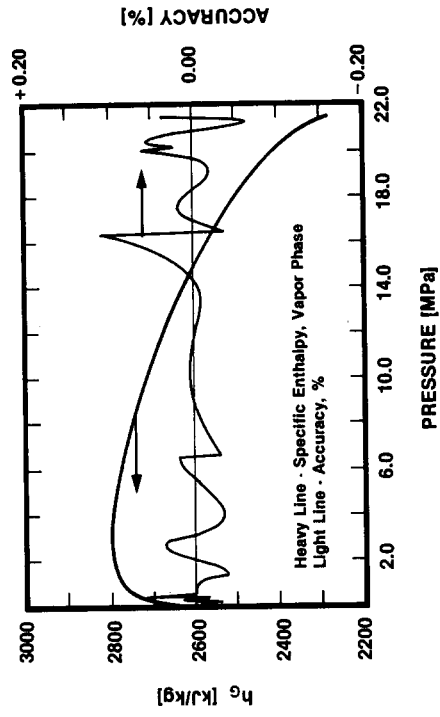


Figure 6. Specific enthalpy of the vapor phase at saturation.

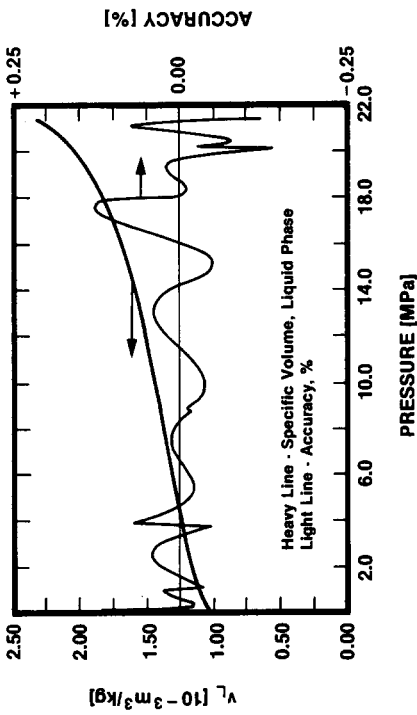


Figure 3. Specific volume of the liquid phase at saturation.

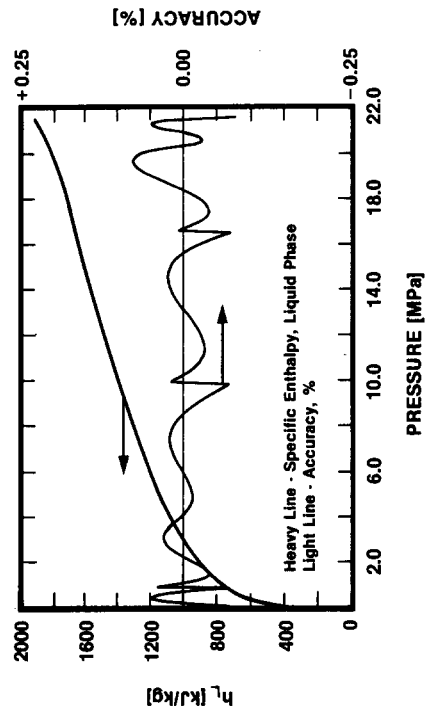


Figure 5. Specific enthalpy of the liquid phase at saturation.

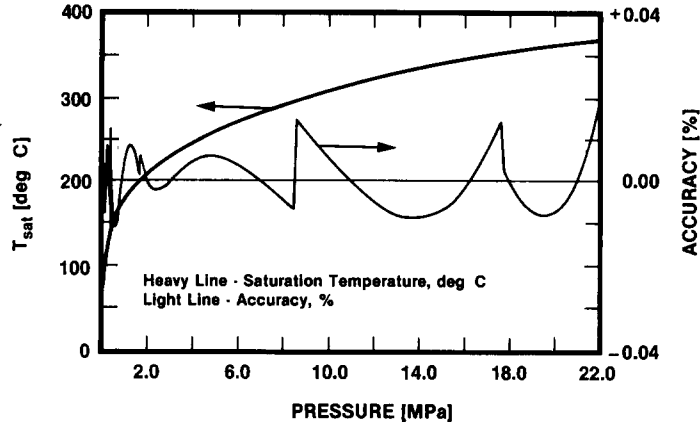


Figure 7. Saturation temperature.

join. Thus the sets of functions for specific volume and specific enthalpy can be used for the calculation of the rate form equation of state (Garland & Sollychin 1988) and in computer algorithms involving the Jacobi of the system matrix. The continuity of the slopes for specific volume (liquid phase), density (gas phase) and specific enthalpy for both phases, is shown in figures 14–17.

The approximation functions can now be applied to the rate form of the equation of state (see the appendix). Plots of the F functions of [A.2] are shown in figures 18–22. Each of the F functions yields a smooth continuous curve, as desired.

Specific volume, liquid phase at saturation

The functions given below are an approximation to the specific volume of light water in the liquid phase, v_L [m^3/kg], for saturation conditions. The pressure range within which they may be used is 0.075–21.5 MPa with the accuracy not worse than 0.14%. Figure 3 shows the accuracy of the approximation.

Approximation functions:

$$v_L = 1.2746977E - 4 * P ** (0.4644339) + 0.001$$

$$0.075 \text{ MPa} \leq P \leq 1.00 \text{ MPa}$$

$$v_L = 1.0476071E - 4 * P ** (0.5651090) + 0.001022$$

$$1.00 \text{ MPa} < P \leq 3.88 \text{ MPa}$$

$$v_L = 3.2836717E - 5 * P + 1.12174735E - 3$$

$$3.88 \text{ MPa} < P \leq 8.84 \text{ MPa}$$

$$v_L = 3.3551046E - 4 * \exp(5.8403566E - 2 * P) + 0.00085$$

$$8.84 \text{ MPa} < P \leq 14.463 \text{ MPa}$$

$$v_L = 3.1014626E - 8 * P ** (3.284754) + 0.00143$$

$$14.463 \text{ MPa} < P < 18.052 \text{ MPa}$$

$$v_L = 1.5490787E - 11 * P ** (5.7205) + 0.001605$$

$$18.052 \text{ MPa} \leq P < 20.204 \text{ MPa}$$

$$v_L = 4.1035988E - 24 * P ** (15.03329) + 0.00189$$

$$20.204 \text{ MPa} \leq P \leq 21.5 \text{ MPa}.$$

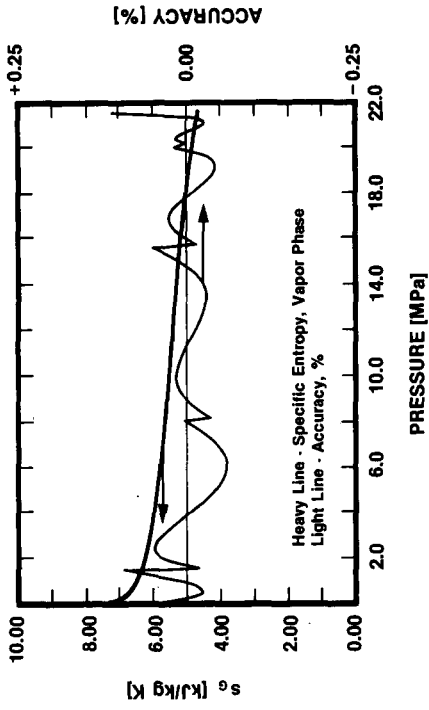


Figure 9. Specific entropy of the vapor phase at saturation.

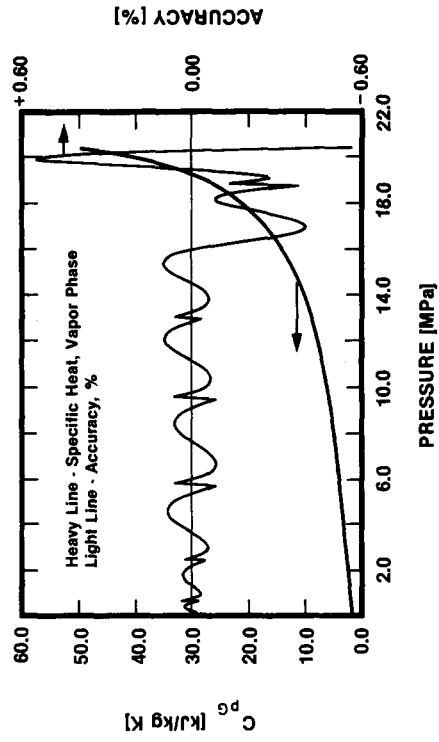


Figure 11. Specific heat of the vapor phase at saturation.

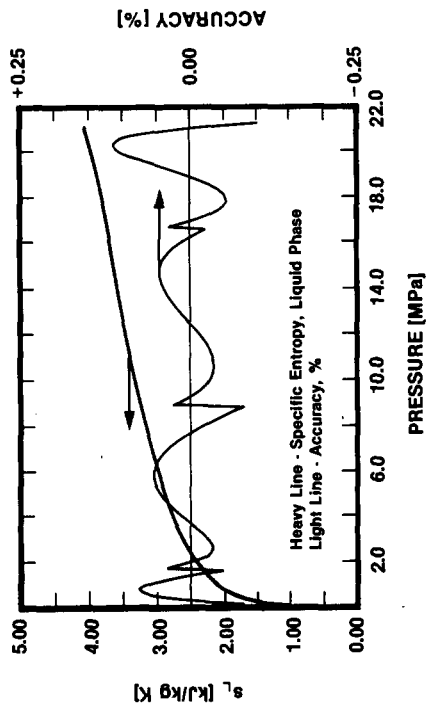


Figure 8. Specific entropy of the liquid phase at saturation.

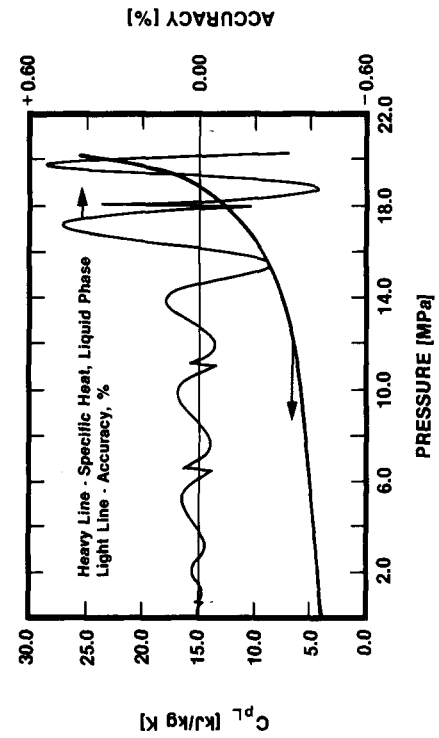


Figure 10. Specific heat of the liquid phase at saturation.

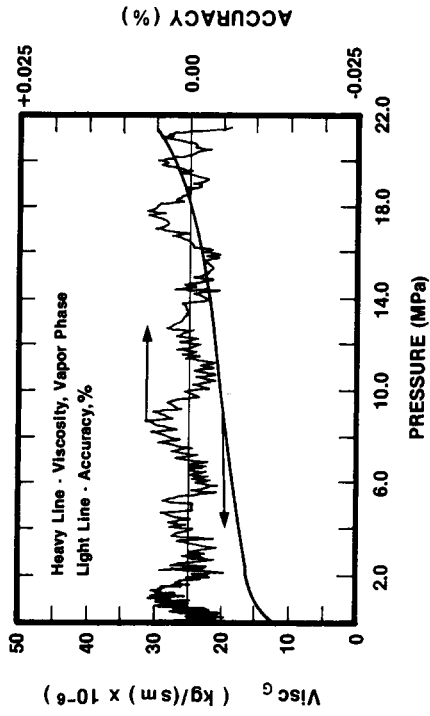


Figure 13. Viscosity of the vapor phase at saturation.

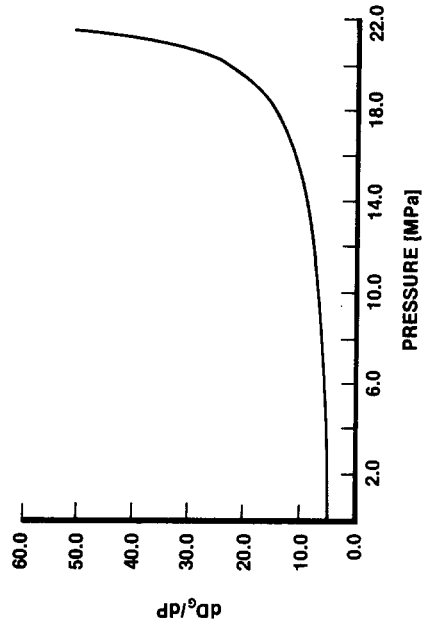


Figure 15. The slope of the vapor density at saturation.

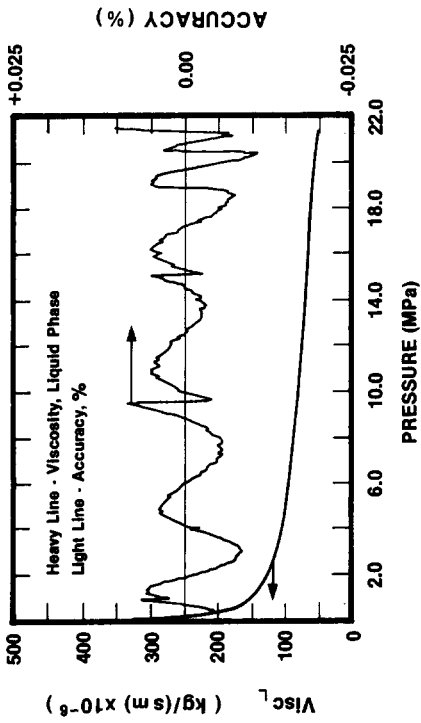


Figure 12. Viscosity of the liquid phase at saturation.

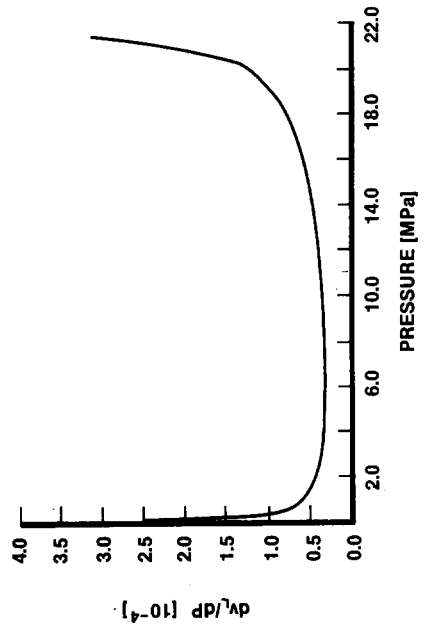


Figure 14. The slope of the specific volume of the liquid phase at saturation.

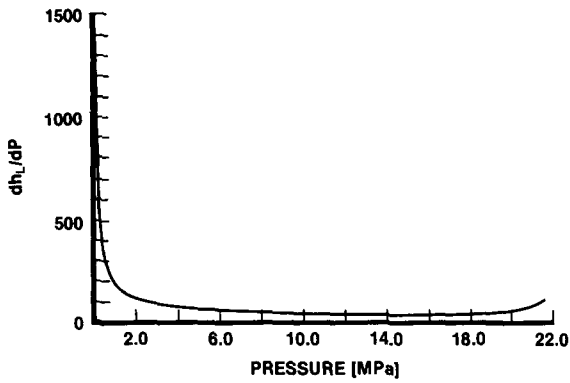


Figure 16. The slope of the specific enthalpy of the liquid phase at saturation.

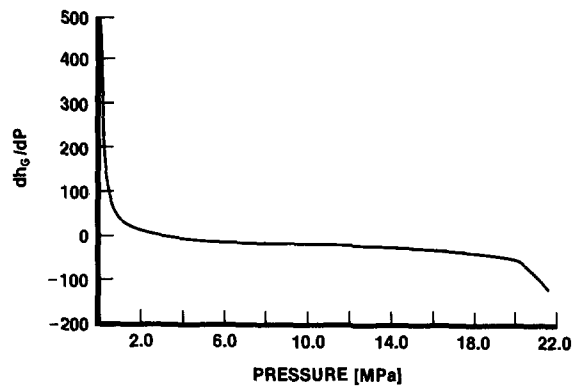


Figure 17. The slope of the specific enthalpy of the vapor phase at saturation.

Density, gas phase at saturation

The following correlations give an approximation to the density of light water in the gas phase, D_G [kg/m³], for saturation conditions. Their range of use is 0.085–21.5 MPa with the accuracy not worse than 0.22%. Figure 4 shows the accuracy of the approximation.

Approximation functions:

$$D_G = 5.126076 * P^{**}(0.9475862) + 0.012$$

$$0.085 \text{ MPa} < P < 1.112 \text{ MPa}$$

$$D_G = 4.630832 * P^{**}(1.038819) + 0.52$$

$$1.112 \text{ MPa} \leq P < 3.932 \text{ MPa}$$

$$D_G = 2.868721 * P^{**}(1.252148) + 3.80$$

$$3.932 \text{ MPa} \leq P < 8.996 \text{ MPa}$$

$$D_G = 0.5497653 * P^{**}(1.831182) + 18.111$$

$$8.996 \text{ MPa} \leq P < 14.628 \text{ MPa}$$

$$D_G = 8.5791582E - 3 * P^{**}(3.176484) + 50.0$$

$$14.628 \text{ MPa} \leq P \leq 18.21 \text{ MPa}$$

$$D_G = 3.5587113E - 6 * P^{**}(5.660939) + 88.0$$

$$18.21 \text{ MPa} < P \leq 20.253 \text{ MPa}$$

$$D_G = 3.558734E - 16 * P^{**}(13.03774) + 138.0$$

$$20.253 \text{ MPa} < P \leq 21.5 \text{ MPa.}$$

Specific enthalpy, liquid phase at saturation

The correlations given below approximate the specific enthalpy of light water in the liquid phase, h_L [kJ/kg], for saturation conditions. The range for which they may be used is 0.075–21.70 MPa with the accuracy not worse than 0.10%. Figure 5 shows the accuracy of the approximation.

Approximation functions:

$$h_L = 912.1779 * P^{**}(0.2061637) - 150.0$$

$$0.075 \text{ MPa} < P < 0.942 \text{ MPa}$$

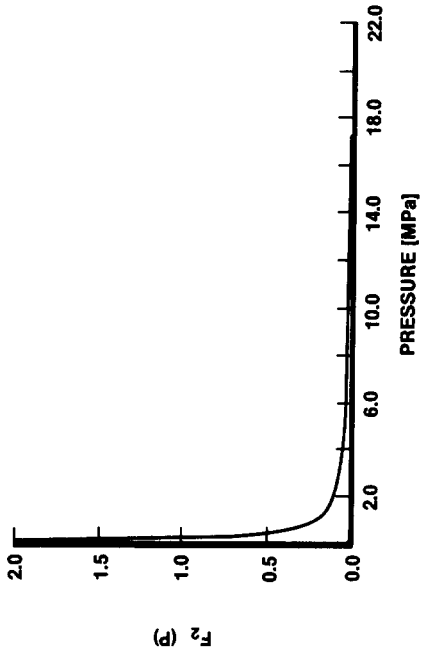


Figure 19. The function F_2 .

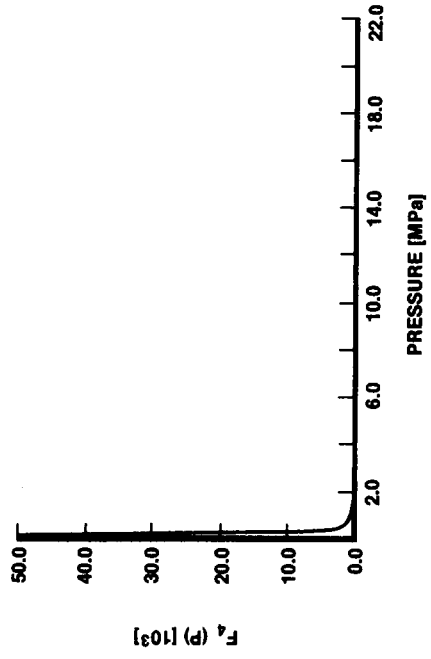


Figure 21. The function F_4 .

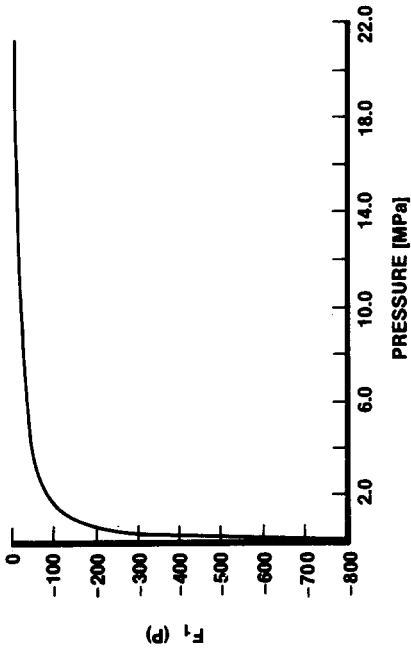


Figure 18. The function F_1 .

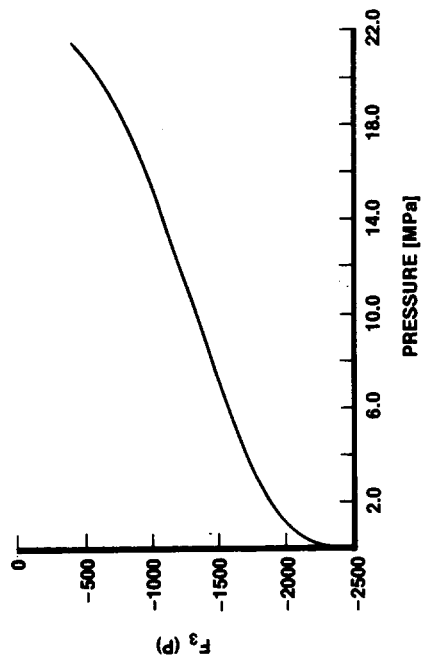
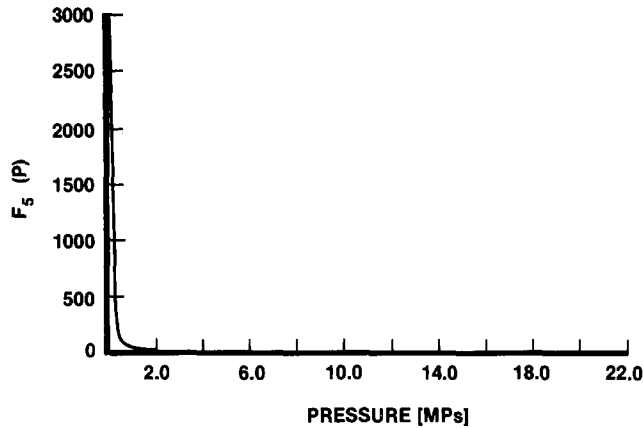


Figure 20. The function F_3 .

Figure 22. The function F_5 .

$$h_L = 638.0621 * P^{**}(0.2963192) + 125.0$$

$$0.942 \text{ MPa} \leq P < 4.02 \text{ MPa}$$

$$h_L = 373.7665 * P^{**}(0.4235532) + 415.0$$

$$4.02 \text{ MPa} \leq P < 9.964 \text{ MPa}$$

$$h_L = 75.38673 * P^{**}(0.8282384) + 900.0$$

$$9.964 \text{ MPa} \leq P < 16.673 \text{ MPa}$$

$$h_L = 0.1150827 * P^{**}(2.711412) + 1440.0$$

$$16.673 \text{ MPa} \leq P < 20.396 \text{ MPa}$$

$$h_L = 9.1417257E - 14 * P^{**}(11.47287) + 1752.0$$

$$20.396 \text{ MPa} \leq P \leq 21.70 \text{ MPa.}$$

Specific enthalpy, gas phase at saturation

The following functions give an approximation of the specific enthalpy of light water in the gas phase, h_G [kJ/kg], for saturation conditions. Their range is 0.075–21.55 MPa with the accuracy not worse than 0.066%. Figure 6 shows the accuracy of the approximation.

Approximation functions:

$$h_G = -4.0381938E - 6 * (3.0 - P)^{**}(15.72364) + 2750.0$$

$$0.075 \text{ MPa} < p \leq 0.348 \text{ MPa}$$

$$h_G = -0.5767304 * \exp(-1.66153 * (P - 3.2)) + 2800.0$$

$$0.348 \text{ MPa} < P \leq 1.248 \text{ MPa}$$

$$h_G = -7.835986 * (3.001 - P)^{**}2.0 + 2.934312 * (3.001 - P) + 2803.71$$

$$1.248 \text{ MPa} < P < 2.955 \text{ MPa}$$

$$h_G = -1.347244 * (P - 2.999)^{**}2.0 - 2.326913 * (P - 2.999) + 2803.35$$

$$2.955 \text{ MPa} \leq P \leq 6.522 \text{ MPa}$$

$$h_G = -0.9219176 * (P - 9.00)^{**}2.0 - 16.38835 * (P - 9.00) + 2742.03$$

$$6.522 \text{ MPa} < P < 16.497 \text{ MPa}$$

$$h_G = -3.532177*(P - 8.00)**2.0 + 29.81305*(P - 8.00) + 2565.00$$

$$16.497 \text{ MPa} \leq P < 20.193 \text{ MPa}$$

$$h_G = -22.92521*(P - 18.0)**2.0 + 44.23671*(P - 18.0) + 2415.01$$

$$20.193 \text{ MPa} \leq P \leq 21.55 \text{ MPa.}$$

Saturation temperature

The correlations given below are for the saturation temperature of light water, T_{sat} [°C]. The pressure range for which they may be used is 0.070–21.85 MPa with the accuracy not worse than 0.02%. Figure 7 shows the accuracy of the approximation.

Approximation functions:

$$T_{\text{sat}} = 236.2315 * P ** (0.1784767) - 57.0$$

$$0.070 \text{ MPa} \leq P < 0.359 \text{ MPa}$$

$$T_{\text{sat}} = 207.9248 * P ** (0.2092705) - 28.0$$

$$0.359 \text{ MPa} \leq P \leq 1.676 \text{ MPa}$$

$$T_{\text{sat}} = 185.0779 * P ** (0.2323217) - 5.0$$

$$1.676 \text{ MPa} < P \leq 8.511 \text{ MPa}$$

$$T_{\text{sat}} = 195.1819 * P ** (0.2241729) - 16.0$$

$$8.511 \text{ MPa} < P < 17.69 \text{ MPa}$$

$$T_{\text{sat}} = 227.2963 * P ** (0.201581) - 50.0$$

$$17.69 \text{ MPa} \leq P \leq 21.85 \text{ MPa}$$

Specific entropy, liquid phase at saturation

The functions given below are an approximation of the specific entropy of light water in the liquid phase, s_L [kJ/kg], for saturation conditions. Their range of use is 0.065–21.25 MPa with the accuracy not worse than 0.12%. Figure 8 shows the accuracy of the approximation.

Approximation functions:

$$s_L = 3.340244 * P ** (0.125474) - 1.20$$

$$0.065 \text{ MPa} \leq P < 1.666 \text{ MPa}$$

$$s_L = 1.748203 * P ** (0.2275611) + 0.40$$

$$1.666 \text{ MPa} \leq P < 8.825 \text{ MPa}$$

$$s_L = 0.2549248 * P ** (0.6381866) + 2.25$$

$$8.825 \text{ MPa} \leq P < 16.66 \text{ MPa}$$

$$s_L = 4.3632383E - 5 * (P - 0.40) ** (3.153273) + 3.50$$

$$16.66 \text{ MPa} \leq P < 21.25 \text{ MPa.}$$

Specific entropy, gas phase at saturation

The following functions give an approximation to the specific entropy of light water in the gas phase, s_G [kJ/kg], for saturation conditions. Their range is 0.025–21.50 MPa with the accuracy not worse than 0.10%. Figure 9 shows the accuracy of the approximation.

Approximation functions:

$$s_G = 6.58681 - 0.335924 * \log(P)$$

$$0.025 \text{ MPa} \leq P \leq 1.48 \text{ MPa}$$

$$s_G = 7.80 - 1.227644 * P ** (0.2481072)$$

$$1.48 \text{ MPa} < P \leq 8.05 \text{ MPa}$$

$$s_G = 6.30 - 0.084638514 * P ** (0.9082161)$$

$$8.05 \text{ MPa} < P \leq 15.64 \text{ MPa}$$

$$s_G = 5.50 - 3.6897161E - 3 * (P - 7.80) ** (2.012466)$$

$$15.64 \text{ MPa} < P \leq 20.00 \text{ MPa}$$

$$s_G = 5.00 - 0.042830642 * (P - 18.7) ** (1.779526)$$

$$20.00 \text{ MPa} < P \leq 21.5 \text{ MPa.}$$

Specific heat, liquid phase at saturation

The correlations given below are an approximation to the specific heat of light water in the liquid phase, C_{pL} [kJ/kg K], for saturation conditions. Their range of use is 0.030–20.3 MPa. For pressures < 13.3 MPa, the accuracy is not worse than 0.08%; for pressures > 13.3 MPa, the error can be as high as 0.60%. Figure 10 shows the accuracy of the approximation.

Approximation functions:

$$C_{pL} = 0.247763 * P ** (0.5704026) + 4.150$$

$$0.030 \text{ MPa} \leq P < 0.671 \text{ MPa}$$

$$C_{pL} = 0.179305 * P ** (0.8967323) + 4.223$$

$$0.671 \text{ MPa} \leq P < 2.606 \text{ MPa}$$

$$C_{pL} = 0.09359843 * P ** (1.239114) + 4.340$$

$$2.606 \text{ MPa} \leq P < 6.489 \text{ MPa}$$

$$C_{pL} = 0.01068888 * P ** (2.11376) + 4.740$$

$$6.489 \text{ MPa} \leq P < 11.009 \text{ MPa}$$

$$C_{pL} = 1.333058E - 4 * P ** (3.707294) + 5.480$$

$$11.009 \text{ MPa} \leq P < 14.946 \text{ MPa}$$

$$C_{pL} = 6.635658E - 3 * (P - 10.0) ** (3.223323) + 7.350$$

$$14.946 \text{ MPa} \leq P < 18.079 \text{ MPa}$$

$$C_{pL} = 4.6844786E - 6 * \exp(0.7396875 * P) + 10.020$$

$$18.079 \text{ MPa} \leq P \leq 20.30 \text{ MPa.}$$

Specific heat, gas phase at saturation

The following correlations give an approximation to the specific heat of light water in the gas phase, C_{pG} [kJ/kg K], for saturation conditions. Their range of use is 0.050–20.40 MPa. For pressures < 16.0 MPa, the accuracy is not worse than 0.12%; for pressures > 16.0 MPa, the error can be as high as 0.60%. Figure 11 shows the accuracy of the approximation.

Approximation functions:

$$C_{pG} = 0.6471635 * (P - 0.006) ** (0.6400569) + 1.90$$

$$0.050 \text{ MPa} \leq P \leq 0.599 \text{ MPa}$$

$$C_{pG} = 0.5560633 * P ** (0.8197355) + 2.00$$

$$0.599 \text{ MPa} \leq P < 2.391 \text{ MPa}$$

$$C_{pG} = 0.3187082 * P^{**}(1.110271) + 2.30$$

$$2.391 \text{ MPa} \leq P < 5.661 \text{ MPa}$$

$$C_{pG} = 0.064275995 * P^{**}(1.766106) + 3.12$$

$$5.661 \text{ MPa} \leq P < 9.458 \text{ MPa}$$

$$C_{pG} = 3.8011048E - 3 * P^{**}(2.816897) + 4.40$$

$$9.458 \text{ MPa} \leq P \leq 12.900 \text{ MPa}$$

$$C_{pG} = 0.1876175 * \exp(0.2466925 * P) + 5.00$$

$$12.900 \text{ MPa} < P \leq 16.309 \text{ MPa}$$

$$C_{pG} = 7.620756E - 3 * \exp(0.4117289 * P) + 9.20$$

$$16.309 \text{ MPa} < P < 18.743 \text{ MPa}$$

$$C_{pG} = 6.5162612E - 6 * \exp(0.756211 * P) + 17.10$$

$$18.743 \text{ MPa} \leq P < 20.40 \text{ MPa}$$

Dynamic viscosity, liquid phase at saturation

The following functions give an approximation to the dynamic viscosity of light water in the liquid phase, Visc_L [10^{-6} kg/m s], for saturation conditions. Their range of use is 0.035–21.45 MPa with the accuracy not worse than 0.10%. Figure 12 shows the accuracy of the approximation.

Approximation functions:

$$\text{Visc}_L = 111.5993 * P^{**}(-0.3425488) + 38.0$$

$$0.035 \text{ MPa} \leq P < 0.960 \text{ MPa}$$

$$\text{Visc}_L = 134.5288 * P^{**}(-0.2848300) + 15.0$$

$$0.960 \text{ MPa} \leq P \leq 3.948 \text{ MPa}$$

$$\text{Visc}_L = 141.5415 - 25.91353 * \ln(P)$$

$$3.948 \text{ MPa} < P < 9.514 \text{ MPa}$$

$$\text{Visc}_L = 113.4599 * \exp(-0.03279562 * P)$$

$$9.514 \text{ MPa} \leq P < 15.074 \text{ MPa}$$

$$\text{Visc}_L = 110.0 - 17.67922 * \exp(0.05556056 * P)$$

$$15.074 \text{ MPa} \leq P < 18.868 \text{ MPa}$$

$$\text{Visc}_L = 9.12152 * P - 0.3159837 * P^{**}2.0$$

$$18.868 \text{ MPa} \leq P < 20.430 \text{ MPa}$$

$$\text{Visc}_L = 64.0 - 0.00261596 * \exp(0.4010038) * P$$

$$20.430 \text{ MPa} \leq P \leq 21.45 \text{ MPa}$$

Dynamic viscosity, gas phase at saturation

The following functions give an approximation of the dynamic viscosity of light water in the gas phase, Visc_G [10^{-6} kg/m s], for saturation conditions. Their range of use is 0.040–21.35 MPa with the accuracy not worse than 0.065%. Figure 13 shows the accuracy of the approximation.

Approximation functions:

$$\text{Visc}_G = 7.473620 * P^{**}(0.2050149) + 7.6$$

$$0.040 \text{ MPa} \leq P \leq 2.207 \text{ MPa}$$

Table 2. Summary of approximation functions

Property	No. of functions	Range of use [MPa]	Worst accuracy [%]
v_L	7	0.075–21.50	0.14
D_G	7	0.085–21.50	0.22
h_L	6	0.075–21.70	0.10
h_G	7	0.075–21.55	0.066
U_L	—	0.075–21.50	0.08
U_G	—	0.085–21.50	0.11
T_{sat}	5	0.070–21.85	0.02
s_L	4	0.065–21.25	0.12
s_G	5	0.025–21.50	0.10
	5	0.030–13.30	0.08
C_{pL}	3	13.30–20.30	0.60
	6	0.050–16.00	0.12
C_{pG}	3	16.00–20.40	0.60
Visc _L	7	0.035–21.45	0.10
Visc _G	6	0.040–21.35	0.065

$$\text{Visc}_G = 3.375163 * P^{**}(0.3916208) + 11.8$$

$$2.207 \text{ MPa} < P \leq 5.480 \text{ MPa}$$

$$\text{Visc}_G = 0.9169410 * P^{**}(0.7644731) + 15.0$$

$$5.480 \text{ MPa} < P < 9.585 \text{ MPa}$$

$$\text{Visc}_G = 5.030544 * \exp(0.5045239 * P) + 12.0$$

$$9.585 \text{ MPa} \leq P < 14.351 \text{ MPa}$$

$$\text{Visc}_G = 0.4423761 * \exp(0.1458726 * P) + 18.8$$

$$14.351 \text{ MPa} \leq P < 81.385 \text{ MPa}$$

$$\text{Visc}_G = 0.01082229 * \exp(0.3071918 * P) + 22.2$$

$$18.385 \text{ MPa} \leq P \leq 20.347 \text{ MPa}$$

$$\text{Visc}_G = 6.6753655E - 6 * \exp(0.6347700 * P) + 25.1$$

$$20.347 \text{ MPa} < P \leq 21.35 \text{ MPa.}$$

SUMMARY

The functions presented in this paper allow rapid calculation of the thermodynamic properties of light water for saturation conditions. Their range of use is from below 1b to just below the critical point with a high enough accuracy for the purpose of thermalhydraulic systems analysis. The ranges and accuracies for each property are summarized in table 2. It should be noted that internal energy, U , can be calculated using the correlations for specific volume and specific enthalpy and the equation $U = H - PV$. This yields an error of $<0.11\%$.

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APPENDIX

The Rate Form of the Equation of State

The development of a non-iterative equation of state for two-phase flow systems was recently investigated (Sollychin *et al.* 1985; Garland & Sollychin 1988). At present, the equation of state is usually solved by iterative numerical techniques. By recasting the equation of state the time derivative of P can be solved directly and the use of iterative methods can be eliminated. The time derivative form of the equation of state was developed by considering an arbitrary volume of two-phase fluid as a thermodynamic system where both phases are at saturation under a uniform saturation pressure, P . By utilizing the total mass of the fluid, M , the total enthalpy in the system, H , the volume of the system, V , and taking the derivatives of these quantities with respect to time, one obtains [A.1] which is dependent on the initial pressure and on the rate of change of mass, volume and enthalpy in the system:

$$\frac{dP}{dt} = \frac{F_1(P)\frac{dM}{dt} + F_2(P)\frac{dH}{dt} + F_3(P)\frac{dV}{dt}}{M_g F_4(P) + M_f F_5(P)}, \quad [\text{A.1}]$$

where

$$F_1(P) = h_G * v_L - h_L * v_G,$$

$$F_2(P) = v_G - v_L,$$

$$F_3(P) = -(h_G - h_L),$$

$$F_4(P) = \frac{dh_G}{dP}(v_G - v_L) - \frac{dv_G}{dP}(h_G - h_L)$$

and

$$F_5(P) = \frac{dh_L}{dP}(v_G - v_L) - \frac{dv_L}{dP}(h_G - h_L).$$

This form involves combinations of the saturation values of specific volume and specific enthalpy in liquid and gas phases, and the derivatives of these properties with respect to pressure. Thus we can incorporate the approximation functions described above to easily solve the equation of state [1]. The derivatives are determined simply by taking the derivatives of the approximation functions with respect to pressure.